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INTERIM
IN-36-CR
6476
P-11

(NASA-CR-199760) RECEIVER DESIGN, PERFORMANCE ANALYSIS, AND EVALUATION FOR SPACE-BORNE LASER ALTIMETERS AND SPACE-TO-SPACE LASER RANGING SYSTEMS Interim Report, 15 Apr. - 15 Oct. 1995 (JHU) 11 p

N96-15753

Unclas

G3/36 0083579

# ELECTRICAL & COMPUTER ENGINEERING

# Interim Progress Report
# on NASA Grant NAG5-2232
## "Receiver Design, Performance Analysis, and Evaluation for Space-Borne Laser Altimeters and Space-to-Space Laser Ranging Systems"

for the period of April 15 1995 to October 15, 1995

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November 1995

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*SUMMARY*

This Interim report consists of a manuscript, "Receiver Design for Satellite to Satellite Laser Ranging Instrument," and copies of two papers we co-authored, "Demonstration of High Sensitivity Laser Ranging System" and "Semiconductor Laser-Based Ranging Instrument for Earth Gravity Measurements." These two papers were presented at the conference *Semiconductor Lasers, Advanced Devices and Applications*, August 21-23, 1995, Keystone Colorado. The manuscript is a draft in the preparation for publication, which summarizes the theory we developed on space-borne laser ranging instrument for gravity measurements.

# Receiver Design for Satellite to Satellite Laser Ranging Instrument

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## 1 Introduction

Precision laser ranging between a host satellite and a co-orbital passive subsatellite can be used to measure the gravity field distribution of the earth. The relative velocity between the host satellite and the subsatellite is a function of the gravity field distributions below the satellites. Precision range measurement can be achieved with a laser ranging instrument on the host satellite and retroreflectors on the subsatellite. Because the divergence angle of the laser beam is much smaller than that of radars, the signal propagation losses in space for a laser ranging instrument is much lower than radars. NASA has studied the use of a laser ranging instrument in the Gravity and Magnetic Earth Surveyor (GAMES) mission [1] [2]. The technique can also be used to map the gravity distribution of the moon and other planets. This paper presents a receiver design and performance analysis of such a laser ranging systems.

The basic laser ranging instrument consists of a laser transmitter, a photodetector, and some signal processing circuitry. Here we consider the use of a semiconductor laser with sinusoidal intensity modulation. The reflected laser beam by the subsatellite is collected by the receiver telescope and then detected by a photodetector. The received signal is then compared with the transmitted signal to determine the amount of phase shift which in turn gives the distance between the two satellites. The higher the modulation frequency, the larger the phase shift in the reflected signal and the finer the ranging resolution. On the other hand, the range measurement is periodic and the period is equal to that of the modulating signal multiplied by the speed of light and then divided by two. One can modulate the laser with several tones at successively higher frequencies to achieve both fine ranging resolution and wide range coverage. The receiver contains a narrow band filter for each tone and there should be no interference between channels. These type of laser ranging systems are relatively easy to built since they are direct detection system.

The two satellites considered for NASA's GAMES mission was 100 to 300 km apart and the relative velocity is expected to be about 1 $m/s$ on average and changes very slowly. It is not practical to use coherent detection scheme in this application because the coherent time of lasers available today are shorter than or comparable to the round trip propagation time between the two satellites (0.67ms/100km). The laser transmitter being considered here are AlGaAs laser diodes at about 800nm wavelength. The advantage of this type of laser diodes are the small size, high efficiency, and relatively high output power. Sinusoidal intensity modulation is preferred because the receiver signal processing are similar to those in conventional Doppler radars and the technologies are mature and readily available.

The photodetectors which render the highest receiver sensitivity are photomultiplier tubes (PMT) and silicon avalanche photodiodes (APD). Both devices can be operated in either photon counting mode or analog mode. In photon counting mode,

a photocurrent pulse generated by a single photon absorption is detected by a discriminator as a discrete event. The use of the discriminator helps to reduce the effect of the photodetector gain randomness as well as the circuit thermal noise. On the other hand, the receiver is narrow band and should reject most of the thermal and photodetector gain noises. Therefore we may just use these photodetectors in analog mode and avoid the time walk introduced by the discriminators itself.

## 2 The Link Equation

We first consider single tone sinusoidal laser intensity modulation. The transmitted laser signal power, $P_x(t)$, can be written as

$$P_x(t) = m_1 P_o cos(\omega_0 t) + P_o \tag{1}$$

with $0 < m_1 \leq 1$ the modulation index, $P_o$ the average transmitted laser power, and $\omega_o$ the modulating frequency. The peak power of the laser is $(1 + m_1)P_o$.

The laser beam propagates to the subsatellite and a small fraction is reflected back by the corner cube. The reflected laser beam is assumed to be diffraction limited with the full divergence angle $\theta_c = 2 \times 1.22\lambda/\phi_c$. with $\phi_c$ the diameter of the corner cube and $\lambda$ is the wavelength. Here we assume the subsatellite has one corner cube on the rear and points to the host satellite. This may be achieved by designing the shape of the subsatellite like a bullet so that its orientation can be kept to a certain degree by the drag of the upper atmosphere. In the absence of atmosphere such as around the moon, the subsatellite can be a sphere covered with a number of corner cubes. We will expand our analysis to include multiple corner cubes in a later section.

The fraction of transmitted signal captured by the receiver as a function of the range can be written as

$$A(z) = \left(\frac{\phi_c}{\alpha_X z}\right)^2 \left(\frac{\phi_r}{\theta_c z}\right)^2 \eta_r = \frac{\eta_r \phi_c^4 \phi_r^2}{5.95\lambda^2 \alpha_X^2 z^4} \tag{2}$$

where $z$ is the range between the two satellite, $\alpha_X$ the transmitted laser beam divergence angle, $\phi_r$ the diameter of receiver telescope, and $\eta_r$ the transmission efficiency of the receiver optics. Notice that the received signal power is proportional to the fourth power of the size of the corner cube and inversely proportional to the fourth power of the distance.

The received optical signal power can be written as

$$P_r(t) = A[z(t)]P_x(t - 2\frac{z(t)}{c}) + P_b \tag{3}$$

with $P_b$ the average background illumination. Note the range is a function of time. For convenience, we choose the time origin equal to the start of the observation interval, i.e.

$$z(t) = z_0 + vt \tag{4}$$

with $z_0$ the range to the subsatellite at the beginning of the observation interval and $v$ the relative speed between the two satellites. It is this relative speed which we have to estimate in order to determine the gravity field. As mentioned earlier, $v$ is small and changes very slowly over time. The observation time interval in (4) is assumed to be small that $v$ can be approximated as piecewise constant. The propagation losses can also be approximated as unchanged over the observation interval. Substitute (4) and (1) into (3),

$$P_r(t) = A(z_0)m_1P_o cos[\omega_0 t + \theta_0 + \omega_D t] + A(z_0)P_o + P_b \tag{5}$$

where

$$\theta_0 = \omega_0\frac{2z_0}{c} \tag{6}$$

$$\omega_D = 2\omega_0\frac{v}{c} \tag{7}$$

The term $\omega_D$ represents the Doppler shift due to the relative motion of the two satellites.

# 3 Receiver Design

The anticipated relativity velocity between the two satellites is only about 1 m/s and changes very slowly. The Doppler shift can be written as two terms

$$\omega_D = 2\omega_0 \frac{\bar{v}}{c} + 2\omega_0 \frac{\Delta v}{c} \tag{8}$$

with $\bar{v}$ and $\Delta v$ the average and the fluctuation of the velocity. The first term in (8) is just a constant frequency shift. The second term carries the information about the gravity field and needs to be determined with high accuracy. As mentioned earlier, it is assumed that $\bar{v} \sim 1m/s$ and

$$2\omega_0 \frac{\Delta v}{c} T \ll 2\pi \tag{9}$$

with $T$ the observation time. The receiver primarily detects the phase of the received signal. The average Doppler shift may be obtained from successive phase measurements over a much longer integration time.

Gagliardi and Karp [3] showed a maximum a posteriori (MAP) phase estimator for an ideal photodetector and a Gaussian phase distribution. It is given as the solution to the integral equation

$$\hat{\theta} = \sigma^2 \int_0^T x(t) tan[\frac{1}{2}\omega_0(1 + \frac{2\bar{v}}{c})t + \frac{1}{2}\hat{\theta}]dt \tag{10}$$

where x(t) is the photon counting process output from the photodetector and $\sigma^2$ is the variance of the phase. The equation for the MAP phase estimator given in (10) may be depicted as a phase tracking loop called a tan-lock loop [3]. However, it is very difficult to implement a tan-lock loop at the frequencies we are interested in ($>$ 1GHz).

The receiver we are considering consists of a narrow bandpass filter and a frequency and phase detector. It is simple in both concept and implementation though not mathematically optimal. The bandwidth of the filter should be as small as possible but sufficient to accommodate the Doppler shift. Asin radio frequency (RF)

receivers, we further down convert the frequency of the received signal to some intermediate frequency (IF) before the phase detection. The narrow bandpass filter and the subsequent phase detector become much easier to build at lower frequencies. It also enables us to use digital signal processing technique for the filtering and the phase and frequency detection. Figure 1 shows the block diagram of the receiver. The IF frequency should be low enough for easy signal processing but high enough to minimize the effect of the 1/f noise of the subsequent circuit. This type of laser ranging receiver has been used by Payne [4] for precision ranging of stationary targets.

There are many kinds of digital phase and frequency detectors. We consider here a relative simple phase estimator given by

$$\tilde{\theta} = tan^{-1}\left\{\frac{Im[Y(\omega_p)]}{Re[Y(\omega_p)]}\right\} = tan^{-1}\left[\frac{\sum_{n=0}^{N} y_n sin(\omega_p n\Delta t)}{\sum_{n=0}^{N} y_n cos(\omega_p n\Delta t)}\right] \tag{11}$$

where $y_n' s$ are the digitized samples of the down converted signal, $Y(\omega)$ is the Fourier transform of the sampled signal, $\Delta t$ is the sampling interval, $N$ is the number of samples in the observation interval, and $\omega_p$ is the predicted frequency of the received signal given by

$$\omega_p = \omega_{IF} + \overline{\omega}_D \tag{12}$$

with $\overline{\omega}_D$ the predicted average Doppler shift and $\omega_{IF}$ the IF frequency.

This phase estimator is similar to the one used by Payne [4] except that the target in our case is moving constantly and the exact frequency of the received signal is unknown. Because the Doppler shift changes very slowly over time, the prediction of the frequency can be very close once the receiver is tracked on. Next, we derive the effect of a small unknown frequency on the phase estimator.

The average received signal can be written as

$$y_0(t) = I_s cos[(\omega_{IF} + \omega_D)t + \theta_0] \tag{13}$$

with $\omega_D$ the unknown Doppler shift and $\theta_0$ a constant phase. The sampled signal

after the A/D converter over a measurement interval can be expressed as

$$y(t) = \sum_{n=0}^{N} y_n \delta(t - n\Delta t) = \left( \sum_{n=-\infty}^{\infty} \delta(t - n\Delta t) \right) \cdot y_0(t) \cdot w(t) \tag{14}$$

where $y_n = y_0(n\Delta t)$ and $w(t)$ is the windowing function given by

$$w(t) = \begin{cases} 1, & \text{if } 0 \leq t \leq T \\ 0, & otherwise \end{cases} \tag{15}$$

The Fourier transform of $y(t)$ is

$$Y(\omega) = \left[ \frac{2\pi}{\Delta t} \sum_{k=-\infty}^{\infty} \delta(\omega - k\frac{2\pi}{\Delta t}) \right] * Y_0(\omega) * W(\omega) \tag{16}$$

where '$*$' denotes convolution.

Since

$$Y_0(\omega) = \frac{I_s}{2} \left\{ e^{j\theta_0} \delta[\omega - (\omega_{IF} + \omega_D)] + e^{-j\theta_0} \delta[\omega + (\omega_{IF} + \omega_D)] \right\} \tag{17}$$

and

$$W(\omega) = T \cdot \frac{sin(\omega T/2)}{\omega T/2} e^{-j\omega T/2} = T \cdot sinc(\omega T/2) \cdot e^{-j\omega T/2} \tag{18}$$

the Fourier transform of the signal can be written as

$$\begin{aligned} Y(\omega) &= \frac{\pi T I_s}{\Delta t} \left\{ e^{j\theta_0} \sum_{k=-\infty}^{\infty} sinc \left[ \frac{(\omega - (\omega_{IF} + \omega_D) - k\frac{2\pi}{\Delta t})T}{2} \right] e^{-j(\omega - (\omega_{IF} + \omega_D) - k\frac{2\pi}{\Delta t})\frac{T}{2}} \right. \\ &+ \left. e^{-j\theta_0} \sum_{k=-\infty}^{\infty} sinc \left[ \frac{(\omega + (\omega_{IF} + \omega_D) - k\frac{2\pi}{\Delta t})T}{2} \right] e^{-j(\omega + (\omega_{IF} + \omega_D) - k\frac{2\pi}{\Delta t})\frac{T}{2}} \right\} \end{aligned} \tag{19}$$

The sample rate is assumed to be fast enough that there is no aliasing between each terms of the summations in (19) and all the $k \neq 0$ terms may be filtered out. The observation time is set to some multiple of the period of the estimated frequency, i.e.,

$$\omega_p T = 2k\pi, \quad k = integer \tag{20}$$

Substituting $\omega = \omega_p$ into (19) and dropping all the $k \neq 0$ terms

$$Y(\omega_p) = \frac{\pi T I_s}{\Delta t} \left[ sinc\left(\frac{\Delta\omega T}{2}\right) e^{-j(\frac{\Delta\omega T}{2} - \theta_0)} + sinc\left(\frac{2\omega_p T - \Delta\omega T}{2}\right) e^{j[\frac{\Delta\omega T}{2} - \theta_0]} \right] \tag{21}$$

where

$$\Delta\omega = \omega_p - (\omega_{IF} + \omega_D) \tag{22}$$

The phase estimator becomes

$$\tilde{\theta} = tan^{-1}\left\{\frac{-sin(\frac{\Delta\omega T}{2} - \theta_0)\left[sinc\left(\frac{\Delta\omega T}{2}\right) - sinc\left(\frac{2\omega_p T - \Delta\omega T}{2}\right)\right]}{cos(\frac{\Delta\omega T}{2} - \theta_0)\left[sinc\left(\frac{\Delta\omega T}{2}\right) + sinc\left(\frac{2\omega_p T - \Delta\omega T}{2}\right)\right]}\right\} \tag{23}$$

Since $\omega_p T = 2k\pi$, $sinc(\omega_p T - \Delta\omega T/2) = -sin(\Delta\omega T/2)/(\omega_p T - \Delta\omega T/2)$

$$\begin{aligned}\tilde{\theta} &= tan^{-1}\left[tan(\theta_0 - \frac{\Delta\omega T}{2})\frac{\frac{1}{\Delta\omega} + \frac{1}{2\omega_p - \Delta\omega}}{\frac{1}{\Delta\omega} - \frac{1}{2\omega_p - \Delta\omega}}\right] \\ &= tan^{-1}\left[tan(\theta_0 - \frac{\Delta\omega T}{2})\left(1 - \frac{\Delta\omega}{\omega_p - \Delta\omega}\right)\right] \end{aligned} \tag{24}$$

If the frequency of the received signal is known, i.e., $\Delta\omega = 0$,

$$\tilde{\theta} = tan^{-1}[tan(\theta_0)] = \theta_0 \tag{25}$$

When the frequency of the received signal is unknown, the estimated phase contains an additional term which is equal to the frequency difference between the predicted and the actual times one half the observation time. The unknown frequency also introduces a bias term in the estimated phase. The relative error in the estimated phase due to this bias is approximately

$$\epsilon_b = \frac{\Delta\omega}{\omega_p - \Delta\omega} = \frac{\Delta\omega}{\omega_{IF}} \tag{26}$$

Therefore, the IF frequency should be much greater than the amount of the unknown Doppler shift. It also seen from (34) that the observation interval has to satisfy $\Delta\omega T/2 \ll \pi$ in order to avoid ambiguity in the phase measurement.

# 4 Receiver Signal to Noise Ratio

The signal output from an ideal photodetector can be modeled as a Poisson random point process with the average counting rate with the average photocurrent given by

[5]

$$I_{pd}(t) = qG\frac{\eta P_r(t)}{hf} + I_d \tag{27}$$

where $q$ is the electron charge, $G$ is the average photodetector gain, $\eta$ is the quantum efficiency of the photodetector, $hf$ is the photon energy, and $I_d$ is the photodetector dark current. The amplitude of the sinusoidal signal is obtained by substituting (5) into (27), as

$$I_s = qG\frac{\eta}{hf}A(z_0)P_o m_1 \tag{28}$$

The total noise output from the photodetector consists of the shot noise associated with the photon detection and the circuit noise. The latter can be modeled as independent and additive Gaussian noise. The former is signal dependent and the exact distribution function is not readily available. However, as shown in [6], the photodetector shot noise can be considered as independent Gaussian noise if the receiver is narrow banded and the signal to noise ratio is high. The one sided noise current spectral density can be written as [7]

$$\begin{aligned} N_n &= 2qG^2F[< I_{pd}(t) > + I_d] + N_{amp} \\ &= 2qG^2F[\frac{\eta q}{hf}A(z_0)P_o + \frac{\eta q}{hf}P_b + I_d] + N_{amp} \end{aligned} \tag{29}$$

where $F$ is the excess noise facto, $N_{amp}$ is the noise density of the preamplifier, and $< I_{pd}(t) >$ is the average DC photocurrent.

If a photomultiplier tube (PMT) is used, $N_{amp}$ can often be neglected because the average photodetector gain is large. The excess noise factor of a PMT is given by $F_{PMT} \approx m/(m-1)$ with $M$ the average gain of each dynode ($m \sim 3-5$). If an APD is used, the excess noise factor is given by [8] $F_{APD} = k_{eff}G + (2 - 1/G)(1 - keff)$ with $k_{eff}$ the ratio of the hole to electron ionization coefficients. The circuit noise has to be considered in this case. The circuit noise consists of primarily the preamplifier noise, which is often given by the manufacture or can be measured directly.

The signal to noise ratio (SNR) at the output of the narrow band filter is given as

$$\begin{aligned} SNR &= \frac{I_s^2/2}{N_n B_n} \\ &= \frac{\frac{1}{2}\left[\frac{\eta}{hf}A(z_0)P_o m_1\right]^2}{2F[\frac{\eta}{hf}A(z_0)P_o + \frac{\eta}{hf}P_b + \frac{I_d}{q}]B_n + \frac{N_{amp}}{q^2G^2}B_n} \end{aligned} \qquad (30)$$

with $B_n$ the one sided noise bandwidth of the filter.